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Distance Sampling to Estimate the Abundance of Birds with Sector and Radial Radar Detection Methods

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Abstract

Distance sampling, a widespread method in ecology, can be applied to estimate seabird abundance from a radar detection image. As the radar was placed on the seashore, the detections were not equally spread in the radar image, but clustered at the side of the sea. Sector and radial method are two primary methods to get samples from the whole image by collecting the data that were detected in the sectors or on the radial lines, and different sub-sampling methods would lead to different errors of the estimations. This paper builds two models respectively on sector data and radial data based on the assumption that the birds are uniformly distributed horizontally in this area, and uses this model to calculate the seabird density with a given example of radar detection as well as to compare the errors between these two methods. The seabird density estimated with sector data is 987 per square kilometer with large errors coming from the nonuniform patterns of birds. While, combined with the deviation mainly caused by the size of birds, the radial method achieved a density of 876 objects per square kilometer. Considering the nonuniform patterns for the sea birds, radial method is the optimal method to give a stable and reliable measurement and estimation the bird abundance, especially with a modified model where the size of bird taken into account. This radial sampling method can substitute to the method that takes all the detections of the radar image into account in estimating seabird abundance.

Keywords: Distance Sampling; Radar Detection; Sector method; Radial method; Nonuniform distribution; Bird size

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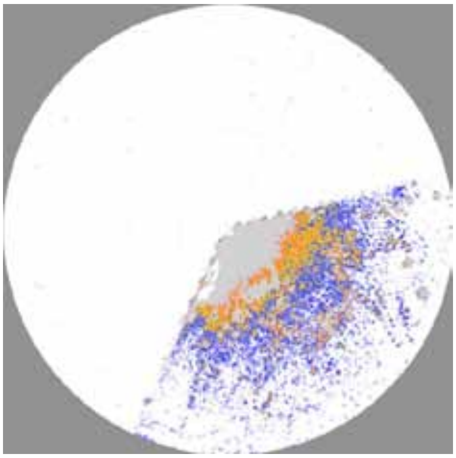
1. Introduction

Distance sampling, usually with line transects or point transects, is a widespread method for estimating animal abundance, including the density of marine birds (Camphuysen 2004). Researchers usually quantify numbers of migrating seabirds with shipboard or aerial line transects sampling.

The commonly used methods to detect marine birds contain visual estimates, optical range finders, binocular reticules, radars, and laser range finders. The method which should be used is determined by the characteristics of the surveyed populations, environmental conditions, and the accuracy required (Mateos 2010). It is difficult to physically watch and map the trajectories of migrating birds as they cross an area of open sea for human observers when the birds change their flight path all the time, thus the biased distance estimates may pose the main problem of the visual estimation of distances and transfer the errors to the estimates of bird abundance. So visual estimates are not a good way to collect and calculate the seabird abundance which makes remote techniques recommended to be used for observing bird behavior and providing robust objective data for modeling seabird density.

Lots of studies (see) were done in the part of estimating marine bird abundance with remote measurement, especially in the case of offshore wind farms on the issue of comparing the changes in bird migration prior to the construction and post to the construction of this wind farm. These remote techniques require a stable platform for mounting and collecting data from this area in order to provide validation of probability models when estimating the number of birds. Some review papers had assessed the degree of usefulness and limitations of different existing remote technologies for studying bird behavior. Radar detection is a highly demanded measurement for the environmental impact studies.

The radar used in this study swept a full circle in ca 2.5 seconds, exporting one radar scanning plot of full circle after one sweeping. By selecting only the “good” images generated during the survey time (usually several months) and computing a global image from this selection image (in this case, 1071 images merge the global image), researchers could get comprehensive information for this study lasting several months with very high sampling frequency from this global radar image. The echoes other than bird detections are then removed from this global image, yielding a total number of ca 8000 observations. An example of processed radar global image is shown in *Figure 1*.



Example of radar global image: birds collected over 4 months colour blue. The grey area indicates that large proportion of birds was detected in this area, and the area appears orange meaning that the proportion of birds detected in this area is less than that of the grey area, but larger than that of the blur area. Almost all the birds are detected within a sector of an angle of *approximately* 130°. *The grey colour also shows around the mid effective detection distance off the radar location except for the area that close to the radar location.*

As this radar was originally placed on the seashore, it is not surprising to find that nearly all the detections in this image are within a sector of angle ca 130° though it actually recorded the full circle. A high proportion of birds were observed close to the radar location which coloured grey in the plot, while low proportion of birds was detected at the edge of the effective detection distance. But it should be noticed that the grey colour also showed around the mid effective detection distance in the centre of detection sector.

As the radar detection usually being considered as point transect sampling, a direct way to deal with this image is to collect all the detection information from this radar image and model with point transect theorem, which is over complex in the process of collecting all points from the whole circle and inaccurate caused by the process faults. Hence, a sampling method should be specified to use to collect points and record information from the global image for estimating the seabird abundance. Sector method, which collected the data within some small angle sectors (to be 2° in this case) randomly selected from the data clustering sector, is one of method that is to be used in this paper besides the radial method, which collected the data detected on the radial lines.

The sampling method should be centred in the area on the side of the sea where most detection are found and provide randomly and uniformly sampling transect in the effective detection place. Taking radial radar as an example, from the approximately semi-circular section of the global image where there are detections, equally spaced radials are randomly selected: the first angle is taken at random and others radials are systematically separated by a fixed angle. In this case study, 56 radials separated by a $2°$ angle were selected, with the first angle being 34°, and the data came from objects on radials of each angle. The radial sampling plot is shown in *Figure 2*.

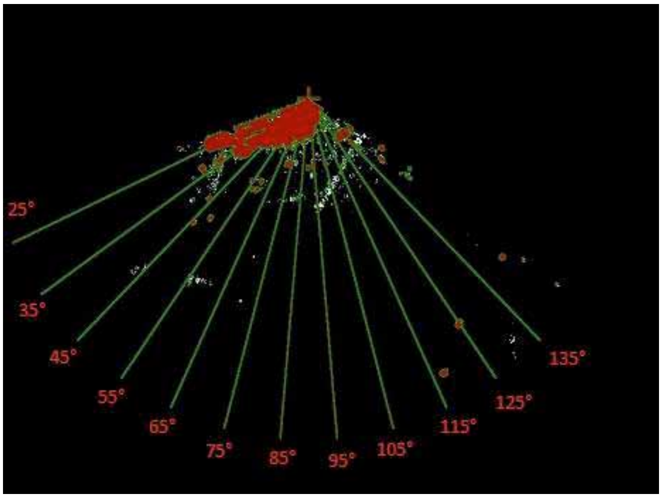


Fig. 1. Example of an image sampled with 12 radials spaced by 10° *angle. All the radial lines are of negligible width. All the birds intersected by the radial line are recorded into the dataset. The bird with larger size is more likely to be picked from the image than the small bird.*

The purpose of this paper is to model the detection capability of a given type of radar across its entire detection range based on the data generated from the previous procedures, specifically, to estimate:

- The first point is to estimate effective detection distance w, knowing the reliable limit of our detections and verifying the effective detection area;

- The second point is to get the detection function $g(\cdot)$, obtaining a correction factor dependent on distances using the frequency of birds detected;
- Then calculate sampling variance $Var(n)$ computed empirically between replicates, estimating detection homogeneity across the study area;
- And get the result of the coefficient of variation $CV(D)$, to estimate the density error.

2 Model Specification

2.1. Model with sector method

The shape of radar detection area appears to be a part of sphere with a centre at the location of radar, thus a model with 3-dimensions detection area could be used to get the birds density of per cubic kilometre. By assuming that the birds are uniformly distributed on horizontal and vertical axis across the radar detection area and the birds are identically independently distributed with time, the density of the bird can be calculated by,

$$D_{sector} = \frac{E(n)}{k \cdot a \cdot P_{a\_sector}} \qquad (1)$$

where k represents the number of sector units for the sector method. And one sector unit covers an area of size a ($a = \delta \cdot \frac{4\pi w^3}{3} = \frac{2}{360} \cdot \frac{4\pi w^3}{3}$), as w is the right truncation distance.

Denote $f(r)$ as the detection density function, which indicates the probability to detect a bird at distance r on the condition that the radar detected n birds in the detection area,

$$f(r)dr = \frac{g(r)}{P_{a_{sector}}} \left[\frac{4\pi r^2 dr}{4\pi w^3/3}\right], \qquad (2)$$

the detection probability of the sector radar P_{a_sector} can be displayed with(3),

$$\hat{P}_{a\_sector} = \frac{3g(0)r^2}{\hat{f}(0)w^3} = \frac{3}{\hat{h}(0)w^3}. \qquad (3)$$

under the condition $g(0) = 1$, after denoting $\hat{h}(0) = \lim_{r\to 0}\left[\hat{f}(r)/r^2\right]$. Therefore the density of seabird can be estimated from,

$$\hat{D}_{sector} = \frac{n\hat{h}(0)}{4\pi\delta k} \qquad (4)$$

The parameters of the detection density function $f(r)$ are estimated by finding the values to the parameters which maximize the likelihood function of detecting n birds in the survey with given distances.

Under the condition that the density of birds is uniformly distributed in the survey area horizontally and vertically, this 3D model would be appropriate, and can also be modified with nonuniform distribution of birds. However, though the estimation from this model is extremely accurate, it would over rely on the model assumptions, and be exceedingly complex to calculate with nonuniform distribution. As the birds cannot uniformly spread in the survey area horizontally and vertically, the 3D model is not fitted for this radar detection, thus another model should be considered.

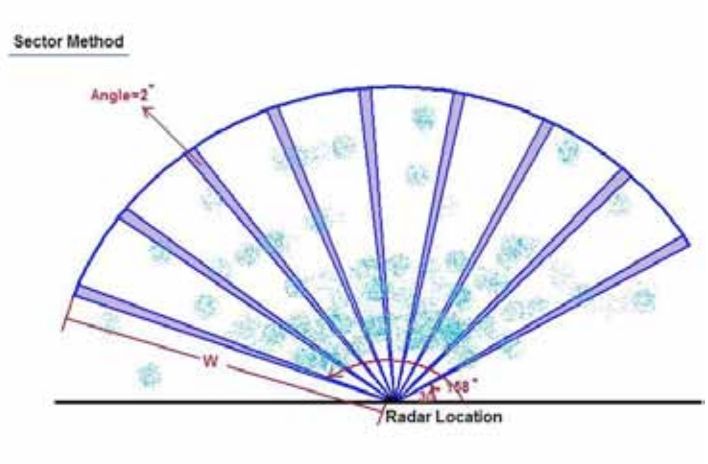


Fig.3. Sketch plot for the sector method. The angle of the sector is 2° *and is separated from the next sector by an angle also equal to* 14°*. The purple areas are the sampled area, where the blue points represents the birds detected in the global detection image.*

As the length of time of recording for the full circle (i.e. 360°) is ca 2.5 seconds, a single detection of the radar can be considered as an instant shoot for the positions of birds. If the number of birds in the detection area is stable and independent about time, considering the mean speed of a bird (ca $10-20\ m/s$) and the fairly large distance the bird has to fly to "escape" the radar, we assume that every bird flying in front of the radar will be detected if within the range of certain detection. Thus, it is reasonable to assume that horizontally the birds flew above the sea are uniformly distributed. In the real case, if the birds were detected very close the radar, these birds would flew above the radar which beyond the radar detection area. So a left truncation of the data is essential. After left truncating the data at certain distance, the sampled birds would be horizontal uniformly distributed. For vertical, taking the fact that different species of birds have their favourite flying height into account, the number of bird increases and then decreases with the flying height grows in the effective detection area. As the height of bird flying grows over a certain distance, the number of birds flying at this height would be very small. By considering the detection probability at this height is relative small, the radar could rarely detect any birds at this very high vertical distance. A reasonable assumption can be built according to previous discussions on that the birds only flying passing the detection area under a certain height, indicating the effective survey area to be more like a cuboid area (one aspect of cuboid is a sector, and the height of the cuboid is the height of under which the birds usually flying by) rather than a sector cone.

Taking r as the horizontal distance between the bird and the sector radar, w as the furthest distance of the birds detected; and using parameter P_{a_sector} to represent the probability to detect a single bird given that it is in the effective survey area, the relationship between the detection density function $f(r)$ and detection function $g(r)$ can be expressed as,

$$f(r) = \frac{2\pi r \cdot g(r)}{\pi\omega^2 \cdot P_{a\_\text{sector}}} \tag{5}$$

Denote $h(0) = \lim_{r\to 0}[f(r)/r]$, the probability to detect n birds in the survey area can be estimated by,

$$\hat{P}_{a\_\text{sector}} = \frac{2g(0)}{\omega^2 \cdot \hat{h}(0)} \tag{6}$$

And if the detection probability assumes to be 1 at distance 0, which is $g(0) = 1$, the detection probability P_{a_sector} can be estimated with $\hat{P}_{a\_\text{sector}} = 2/[\omega^2 \cdot \hat{h}(0)]$

The sampling unit contains k sectors (in this case, $k = [(158 - 30)/4] + 1 = 33$) and one sector cover a horizontal area of a ($a = \delta\pi w^2 = \frac{2}{360}\pi w^2$). Under the assumptions that the density of bird is independent about time, the density of the area can be expressed as,

$$D_{sector} = \frac{E(n)}{k \cdot a \cdot P_{a\_sector}} \tag{7}$$

The radar captured n numbers of birds during the survey time, and the detection probability P_{a_sector} can be estimated by $\hat{P}_{a\_\text{sector}} = 2/[\omega^2 \cdot \hat{h}(0)]$. Therefore, the estimating equation for the density of the area is,

$$\hat{D}_{sector} = \frac{n \cdot \hat{h}(0)}{2k \cdot \delta\pi} \tag{8}$$

The detection density function $f(r)$ is modified using MLE method. As defining

$$\mathcal{L}(\theta) = \prod_{i=1}^{n} f(r_i) \tag{9}$$

Where r_i is the ith recorded distance, $i = 1, \ldots, n$. The log-link of the likelihood function can be expressed as;

$$l = \log_e[\mathcal{L}(\theta)] = \sum_{i=1}^{n} \log_e[f(r_i)] \tag{10}$$

If we set $f(r) = r \cdot g(r)/A$, where $A = \int_0^w r \cdot g(r)\ dr$ is constant to r, then the log-link function l can be expressed using the detect function $g(r)$;

$$l = \sum_{i=1}^{n} \log_e\left[\frac{r_i \cdot g(r_i)}{A}\right] = \sum_{i=1}^{n} \log_e[r_i \cdot g(r_i)] - \text{n}\log_e A \tag{11}$$

So the parameters of the detection density function $f(r)$ which also are the parameters of the detection function $g(r)$ can be calculated by the modifying the log likelihood l to be the smallest. We can use normal and hazard-rate function to model the detection function.

On the assumptions that;

[1]. The sectors were chosen by a randomly selected starting angle.
[2]. All the birds in the selected sectors are recorded in the data set and every data shown in the data set is a true observation of seabird.
[3]. Birds which flew very close to the radar location are certain to be detected.
[4]. The birds are uniformly distributed horizontally in the survey area, or on the side of sea;
[5]. The birds only flying passing the detection area under a certain height, and
[6]. The density of the birds is stable but independent with time and location, so the data is identical independent distributed with each other inside the sector along with the case across the sectors.

The density of the birds on the detection area (or flying above the sea) is,

$$\hat{D}_{sector} = \frac{n \cdot \hat{h}(0)}{2k \cdot \delta\pi}, \tag{12}$$

with the variance of the detection can be expressed as

$$\widehat{Var}(\hat{D}_{sector}) = \hat{D}_{sector}^2 \left\{\frac{\widehat{Var}(n)}{n^2} + \frac{\widehat{Var}(a \cdot \hat{P}_{a\_sector})}{(a \cdot \hat{P}_{a\_sector})^2}\right\} \tag{13}$$

where $\widehat{Var}(n)$ is calculated using the variance of the number of birds for each sector.

2.2. Model with radial method

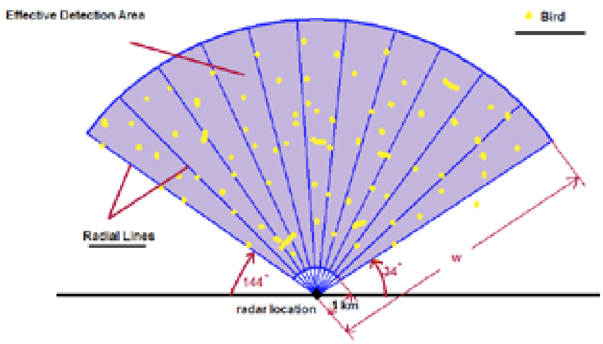


Fig.4. Sketch plot for the radial method sampled with 12 radials spaced by 10° angle starting from the angle of 34°*. The width of the radial line is L. In this case,* $L = 30\ cm$*, which is relative small comparing to the distance between birds and the radar. And the effective detection area for one radial is assumed to be a sector area with angle* $\varepsilon \cdot 360^{\circ}$ *and radius* w*.*

A 3D model could be also applied to this radial method. But similarly with sector radar, under the assumption that the birds only flew by the survey area (seashore) under a certain height H, a model can be built based on 2D case. Considering the birds detected beyond a certain distance (left truncation), they are uniformly distributed horizontally and thus are uniformly sampled from the radial lines. The width of each radial line is $30\ cm$, which is relative small comparing to the scale of the distance and the diameter of birds in the image. Therefore the probability to sample a bird by the radial lines from the radar image is only decided by the diameter of the bird, the number of radial lines existing in the sampling area and the distance of the bird off the radar location. Denoting $2v$ as the diameter of the bird in the radar image, k as the number of radial lines existing in the sampling area, and ε as rate of the effective sampling area against the whole detection area (in the given case, the radial lines starts at the angle of 34°, and ends at the angle of 144°, thus, $\varepsilon = 110/360$), the probability to sample a bird by the radial lines at distance r is;

$$P_1 = P_r\{\text{a bird with size } v \text{ intersected the radial line at the distance } r\} = \frac{2kv}{\varepsilon 2\pi r}$$

The probability of detecting a bird with size v given that the bird is in the area of $(r, r+dr)$ on the radial line is,

$P_2 = P_r\{\text{birds detected}|\text{the bird in } (r, r+dr) \& \text{intersected the radial line}\} = g(r)$.

And for the probability of a bird being in a radius of $(r, r+dr)$ is $P_3 = \varepsilon 2\pi r dr/[\varepsilon \pi w^2]$, the detection density function $f(r)$ for the sampled data can be expressed as,

$$f(r)dr = \frac{P_2 \cdot P_1 \cdot P_3}{P_{a\_radial}} = \frac{g(r)2kv}{P_{a\_radial}} \cdot \frac{dr}{\varepsilon \pi w^2} \quad (14)$$

From the previous equation, we get to know that, the size of the bird would affect the likelihood to detect the bird at a given distance by the radial lines. For calculating the probability to detect a single bird given the bird is in the survey area P_{a_radial}, here make an assumption that all the birds flying by the survey area were of the same size. That means we ignore the differences of the size of the marine birds, and get the equation for P_{a_radial},

$$\hat{P}_{a\_radial} = \frac{2kvg(r)}{\varepsilon\pi w^2 f(r)} = \frac{2kv}{\varepsilon\pi w^2 \hat{f}(0)} \tag{15}$$

Setting the probability of detecting a bird at distance $r = 0$ is 1 ($g(0) = 1$). The detection density function $f(r)$ is also modified using MLE. Defining

$$\mathcal{L}(\theta) = \prod_{i=1}^{n} f(r_i) \tag{16}$$

Where r_i is the ith recorded distance, $i = 1, \dots, n$. Under the previous assumption that all the birds in survey area are of the same size v, denote the density function $f(r) = g(r)/A$, where A is constant with r, and $A = \int_0^w g(r)\ dr$. The log-link likelihood function equals to

$$l = \log_e[\mathcal{L}(\theta)] = \sum_{i=1}^{n} \log_e \left[\frac{g(r_i)}{A}\right] = \sum_{i=1}^{n} \log_e g(r_i) - \mathrm{n}\log_e A \tag{17}$$

To find the parameters of the density function is to find the numeric values of these parameters that make the log-link l smallest under the condition of given data. With an estimation of $f(0)$, the density function could be modelled. As the effective detection area is $a = \varepsilon\pi w^2$, the density of the birds can be written as;

$$D_{radial} = \frac{\mathrm{E}(n)}{a \cdot P_{a\_radial}}$$

Replacing the detection probability as $\hat{P}_{a\_radial} = 2kv/\left[\varepsilon\pi w^2 \hat{f}(0)\right]$, the estimation of the bird density can be expressed as follows,

$$\hat{D}_{radial} = \frac{n\hat{f}(0)}{2kv} \tag{18}$$

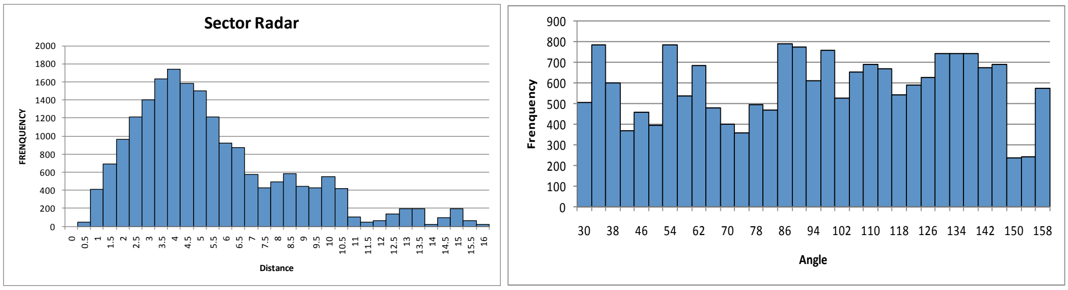
The covariance of the density is;

$$\widehat{Var}\left(\hat{D}_{radial}\right) = \hat{D}_{radial}^2 \left\{\frac{\widehat{Var}(n)}{n^2} + \frac{\widehat{Var}\left(a \cdot \hat{P}_{a\_radial}\right)}{\left(a \cdot \hat{P}_{a\_radial}\right)^2}\right\} \tag{19}$$

Model Application and Improvement

2.3. A sample of sector method

In the sector method, the angle of the sector is 2° and is separated from the next sector by an angle also equal to 2° . In this special case, this means the coverage of 50% of the study area and $[(160 - 30)/360] \cdot 0.5 = 18\%$ of a full circle, since the study area is limited by the shoreline and allows for a range of recording of 130°. A total number of 19253 birds were detected by this sector method.



2 *The histogram of the bird frequency classified by the distance (a) and sector angle (b) for the sector method. The number of detected seabirds first increases then decreases as the detection distance goes up within* $r = 7.5\ km$, *but fluctuates after this distance. The number of birds detected varied between 200 and 800 by sector. The large variability in the second plot is probably because birds often occur in large flock*

As we discuss in section 2, a left truncation of the data is necessary for this model. Judging from the first plot in *Figure 5*, we set this left truncation to be $0.5\ km$ as the detections under $0.5\ km$ are fewer than should be. The first plot in *Figure 5* also indicates that the density function has a similar pattern as the point transect density function except for the data over $r = 7.5\ km$. This unusual fluctuation over the distance $r = 7.5\ km$ is contrary to the form of the detection density function $f(r)$ and reflects the bird flocks at the center of the effection detection sector in the global radar image. In the model implementation, we try the model on the data set which is right trancated at $r = 7.5\ km$ and left trancated at $r = 0.5\ km$ besides on the data set that only left trancated at $r = 0.5\ km$, and compare the two results to give a better estimation of the bird density.

Based on the density equations of sector method, we can use point transect section in Distance software to estimate the density of birds for this sector data. Comparing the log-link likelihood function of the sector radar with that of point transect, we find that evaluation of the sector radar can be substituted by point transect methodology. Considering a point transect sampling with k independent point transects of a same radius w, and taking the data for each sector to the corresponding point transect, the calculation of the sector radar detection density function $f_{\_sector}(r)$ is the same as that of the newly set up sector transect $f_{\_point}(r)$, meaning $f_{\_sector}(r) = f_{\_point}(r) = f(r)$. So the sector radar detection probability P_{a_sector} equals to the results of the point transect detection probability P_{a_point} which we achieved by Distance software under the previous set up. Then as the birds density of the point transect in the Distance software is calculated by $\widehat{D}_{point} = n/\left[k \cdot \pi w^2 \cdot \widehat{P}_{a\_point}\right]$, we get,

$$\widehat{D}_{sector} = \frac{n}{k \cdot \delta \pi w^2 \cdot \widehat{P}_{a\_point}} = \frac{\widehat{D}_{point}}{\delta}$$

where in this case $\delta = 2/360$. Thus, we estimate the density by $\widehat{D}_{radial} = 180 \cdot \widehat{D}_{point}$.

The variance of the sector radar can also be achieved by the variance of the point transect data.

$$\widehat{Var}\left(\widehat{D}_{sector}\right) = \widehat{D}_{sector}^2 \left\{\frac{\widehat{Var}(n)}{n^2} + \frac{\widehat{Var}\left(a \cdot \widehat{P}_{a\_sector}\right)}{\left(a \cdot \widehat{P}_{a\_sector}\right)^2}\right\} = \widehat{D}_{sector}^2 CV\left(\widehat{D}_{point}\right)$$

In the equation, parameter r refers to the horizontal distance between the sector radar and the bird detected; while in the survey, we only have the direct distance between those two subjects. To amend this problem, we assume that for most of the birds we detected, the height of the bird flying is relative small compared with the distance between the bird and radar. Therefore, it is sufficient to replace the unknown horizontal distance with the known direct distance after right truncating the data at $r = 0.5\ km$.

The global radar image is a combined image for four month detection, which means the density that being calculated by the samples of the image is a density of marine bird in this area lasting for four month. It is a cumulated density, but not an instant shoot density. As the number of times that the radar moved in this four month is not provided in this case, we can only display this cumulated density to give a general results of the bird movements over this four month.

We considered six models, first three models (M_1, M_2, M_3) using Normal detection function especially with cosine, Simple polynomial, and Hermit polynomial adjustment terms to each model to build the detection function $g(r)$; other three models (M_4, M_5, M_6) using Hazard-rate detection function and separately those three adjustment terms. Applying these six models to the two sets of data (data set [1] with only left truncation and data set [2] with both left and right truncation), obtain the result, after numerical maximization of the appropriate likelihoods, and then list the relevant estimators in Table 1.

Table 1. The results for the sector method

Data	g(r)	Adjustment	Model	AIC	D	D CV	P
SET [1] Left truncation (0.5 km)	Normal	Cosine	M_1	91657.29	1101.703	1.280264	0.118242
		Simple polynomial	M_2	92030.08	1163.556	1.280066	0.111957
		Hermite polynomial	M_3	92010.71	1203.902	1.28008	0.108205
	Hazard-rate	Cosine	M_4	91790.44	925.9538	1.28007	0.140685
		Simple polynomial	M_5	91763.45	940.4158	1.280086	0.138522
		Hermite polynomial	M_6	91770.48	939.2933	1.280158	0.138687
SET [2] Left truncation (0.5 km) & Right truncation (7.5 km)	Normal	Cosine	M_1	75370.26	958.8937	1.241884	0.319601
		Simple polynomial	M_2	75336.21	1002.158	1.241154	0.305803
		Hermite polynomial	M_3	75538.09	1214.347	1.36536	0.252369
	Hazard-rate	Cosine	M_4	75341.32	1005.2677	1.241625	0.298789
		Simple polynomial	M_5	75335.33	977.4385	1.24112	0.327571
		Hermit polynomial	M_6	75332.69	950.4423	1.241288	0.311086

***D** represent the density of bird, which is calculated using the results of Distance software. **D CV** is the coefficients of variation of the density estimator and **P** is the detection probability of this survey. The model identified as red is the optimal model for the relevant data set diagnosed by AIC statistics.*

From Table 1, we get AIC statistics to be smallest with model M_1 on data set [1], and with model M_6 on data set [2], which indicates model M_1 and M_6 are the optimal models for data set [1] and data set [2] respectively. The bird density achieved with data set [1] is 1101.7, and is 950.4423 with data set [2], being smaller than that of data set [1]. This difference between two results may be caused by the thick tile of the bird detection at long distance in this detection density function, meaning that the birds actually detected at long distance are more than what we expected based on model assumptions. This may caused by large flocks flying past at those distance. But considering the density is a cumulated density that reflects the birds movements over four month, another possible interpretation might be that there are some colonies for marine birds feeding and gregariously living around certain distance away from the seashore, leading to the density of birds to be larger in these areas on the sea than that close to the seashore, which let more number of birds to be detected in these areas. This nonuniform distribution breaks our previous assumptions for the model and makes the detection density tails high.

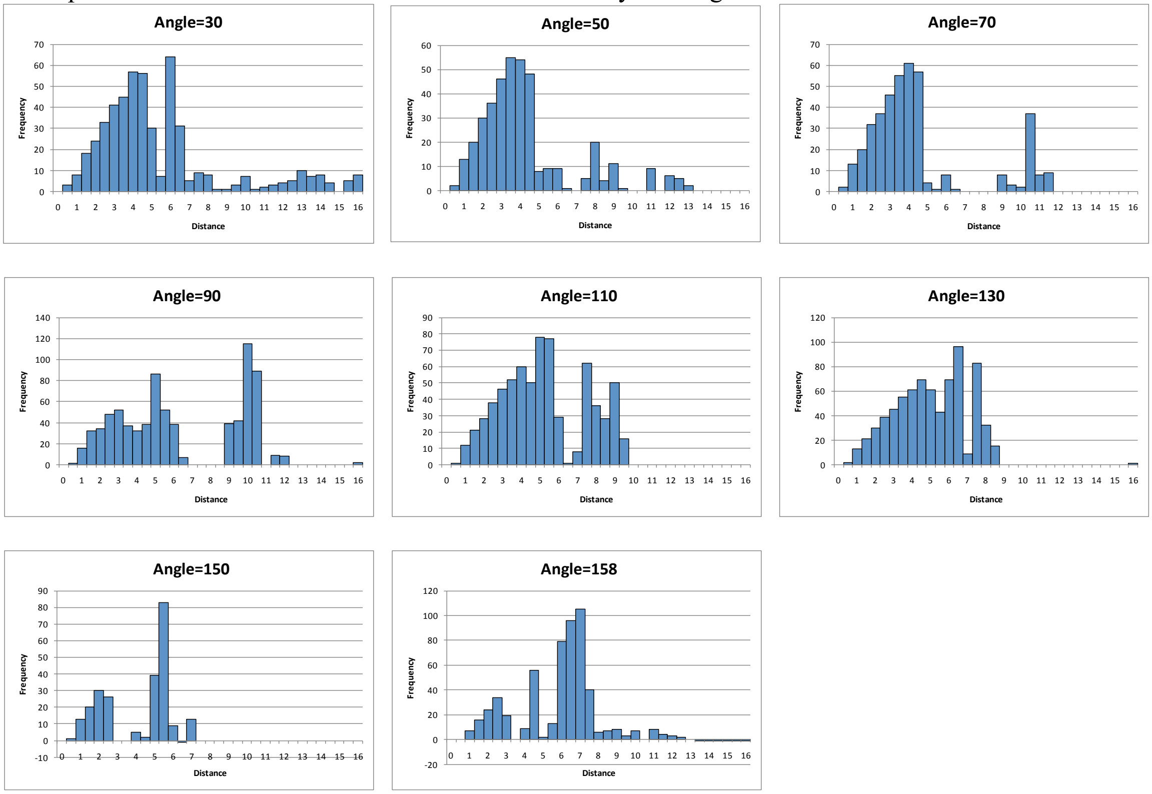
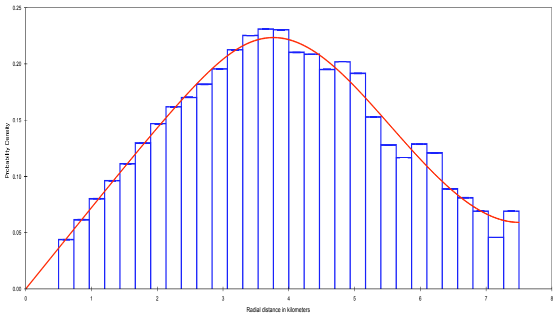
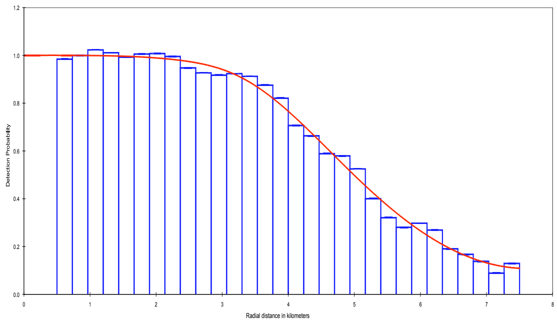


Fig. Histograms of distance with sector method on the angle 30, 50, 70, 90, 110, 130, 150 and 158. The number of birds makes a sudden growth at the distance between 7.5 km *and* 12 km *at angle 50, 70, 90 and 110, which is also shown in the global image as the mid distance grey area in Figure 1. And the distance histogram pattern of angle 150 is totally different with all the others, though no histograms in this figure shows as same as another.*

We can check this interpretation by the field study, but the individual distance histogram of single sector in *Figure 6* can roughly prove it as well. From these plots we find that the detection of birds which abnormally booms high around distance 7.5 km to 10 km is not caused by an anomalous case of one sector, for four out of eight plots in *Figure 6* show an unusual pattern at these distances We can speculate the favourite feeding place for the seabirds (or other bird cluster behaviour) is to be 7.5 km to 10 km off the shoreline. Then for small and large angle detection sector (for example angle 30, 50, 150 and 158), the detection booming could not happened within the radar image as the cluster points at this angle is beyond the radar detection ability. For the middle-angle sectors this clusters cause the sudden booming bird abundance within the distances from 7.5 km to 10 km. The histogram for angle 150 may just caused by special topography of this area.

If we persist with our previous assumptions, a right truncation the data at $r = 7.5\ km$ is necessary. With this truncation, we actually consider the effective detection area within 7.5 km away from the radar or ignore the different patterns for the abundance of seabirds. So the result given by data set [2] is a negative bias estimator for the density of seabirds. We can also change our previous assumption of the uniform horizontal distribution with some other nonuniform distribution by prior studying the regularity of seabird habitat. Here in this case, under the condition of not considering the horizontal density changes by the habitat of marine birds, or only call for the density within distance $7.5\ km$ we choose the result from data set [2] as our estimation of bird density by the sector data. The density of the marine birds in the radar survey area is 985.1405 objects per square kilometre, and the coefficient of variance of this estimation is 1.241. The detect probability function $g(\cdot)$ and density function $f(\cdot)$ estimated using Distance are shown in *Figure 7*.

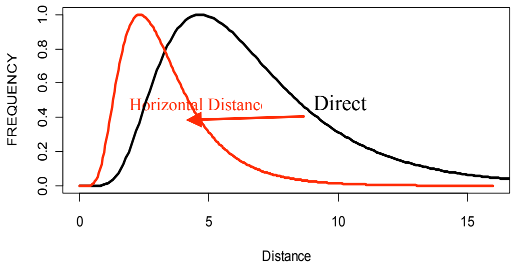




3 *The plot for detection density function $f(\cdot)$ and the detective function $g(\cdot)$ for the sector method. These two functions are modelled with a data set that right truncated at $7.5\ km$ and left truncated at $0.5\ km$ based on the sector method estimation equations. And they fit well within these distances judging from the plots.*

Except the situation discussed above, other condition may also cause a biased result for the density. We discuss some outstanding systematic biased below;

In the equation $\widehat{D}_{sector} = \frac{n}{k \cdot \delta \pi w^2 \cdot \hat{P}_{a\_line}}$, we consider k sectors as identically independent distributed detection areas. But from *Figure 6,* we find that the distance histograms are not of same pattern with different angles. Angle 70 and angle 90 seems to be similar but different with that from angle 110 and angle 130, while plot for angle 150 is totally different with all the others. As the global plot is an average plot for four month detection, this cannot be caused by single migration of birds, thus it is more like to be caused by large flock of birds being detected on occasion or different regions of seabirds inhabiting. This dissimilarity of different angles raises the variance of the estimation, which can be diminished by simply delete the sectors which has the patterns different with all the others (for example angle 150). This removal treatment would cause lots of information lost and would also bias our results. Or we can consider the sectors separately, and get the distribution habitats of seabirds around the whole area.



4 *The sketch plot for scale frequency of birds vs. direct distance and horizontal distance. The black lines shows the detection density pattern with direction distance which we use in the model. The red line represents the pattern of density with horizontal distance which we should use based on the model. The detection probability from the direction distance is larger than that from horizontal distance.*

In the calculation of the density using Distance software, we treat the given distance as the horizontal distance used in the equation, which leads to a positive biased result of the density. For the birds detected close to the radar, the horizontal distance is shorter than the direct distance, and as the birds detected being further from the radar, this horizontal distance gets closer to the direct distance we used, the true detection density function should be right shifted from the detection density function we achieved from the software (*Figure 8*), which makes the true density to be smaller than the density we calculate. There are two ways to amend this problem. One is to do the model with the horizontal distance in the first place if we can get this data from the radar system (this is possible for most of the radars). Another method is to left truncate the data at longer distance. But here in this model as we right truncate the data at 7.5 km which left us few information if we applied large left truncation to the model, losing too much information. So we cannot amend this problem simply by these two ways in this survey, which is also fine because the bias caused by this reason is relevant small.

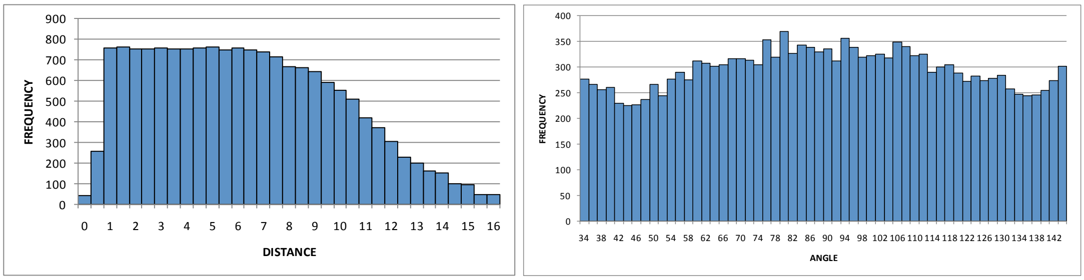
From these discussions above, though we get an estimation based on the sector method, the estimation got huge limitation in accuracy coming from the nonuniform distribution of birds. We then consider the estimation of radial method, and make a comparison with the sector method.

3.1. A Sample of radial method

For the radial method, the width is indeed fairly small, since the width of a pixel that we estimated of about is 30 cm in the real distance scale, which can be considered to be zero in the calculation. From the approximately semi-circular section of the global image where there are detections, equally spaced radials are randomly selected: the first angle is taken at random and others radials are systematically separated by a fixed angle. In this case study, 56 radials separated by a 2° angle were selected, with the first angle being 34°, and the data came from objects on each angle. And a total of 16611 birds were captured in this data set.

Figure 9 shows the histogram of the distance and the sector angle of this case. From histogram of distance we find that the number of birds decreases after the distance 1 km, but increases shortly before distance 1 km. Similar with the sector radar, we should to left truncate the data at $r = 1\,km$ in the calculation. However, for the bird number keep the trend to decay in the plot, we right truncated the data just at the furthest detection distance.

Both the radial method and the sector method are coming from a same radar detection image. So the nonuniform distribution would also affect the samples of the radial methods (see Figure 9). However, flocks would have a smaller effect on these plots because for the radial method, only the small number of birds from the flock that are intersected by the radial would be recorded, whereas for the sector method, the entire flock would be recorded if it falls entirely in the sector.



5 *The histograms of frequency of birds classified by detection distance and radial angle. The amount of birds detected grows within the distance* $r = 1\ km$*, and then shows a trend of declining beyond* $1\ km$*. A fluctuation of bird abundance also appears in the radial angle histogram, but the abundance can be considered stable for the amplitude is relative small.*

We use the line transect section in the Distance software to evaluation the detection density function $f(r)$ given the equation (16) and (17) .We consider the line transect sampling with k independent line transects having length $L = 1\ km$ and width w . Treat the data at each angle of the radial line as the data from each line transect. Thus the estimated function for the radial radar detection density function $f_{\_radial}(r)$ is consist with that of our newly set up line transect detection density function $f_{\_line}(r)$, indicating $\hat{f}_{\_radial}(r) = \hat{f}_{\_line}(r) = \hat{f}(r)$. Use the estimation of the density function, we can given the value of the detect probability;

$$\hat{P}_{a\_radial} = \frac{2kv}{\varepsilon\pi w^2 \hat{f}(0)}$$

In this case, the number of the radial lines is $k = 1 + (110/2)$. And $\varepsilon = 110/360 = 0.305$, w is set to be 16km, v is set to be 0.015. As the detection probability of the line transect which we set in the Distance software is calculated by $\hat{P}_{a\_line} = 1/w\hat{f}(0)$, the P_{a_radial} can be estimated using the result of the P_{a_line}, which is

$$P_{a\_radial} = \frac{2kv}{\varepsilon\pi w} P_{a\_line} = 0.109 \cdot P_{a\_line}$$

Then the density of the radial method can also be estimated from,

$$\hat{D}_{radial} = \frac{\hat{D}_{line}}{v} = 67 \cdot \hat{D}_{line}$$

The coefficient of variance on the estimator of bird density for the two cases is the same,

$$\widehat{CV}\left(\hat{D}_{radial}\right) = \widehat{CV}(n) + \frac{\widehat{Var}\left(a \cdot \hat{P}_{a\_radial}\right)}{\left(a \cdot \hat{P}_{a\_radial}\right)^2} = \widehat{CV}\left(\hat{D}_{line}\right)$$

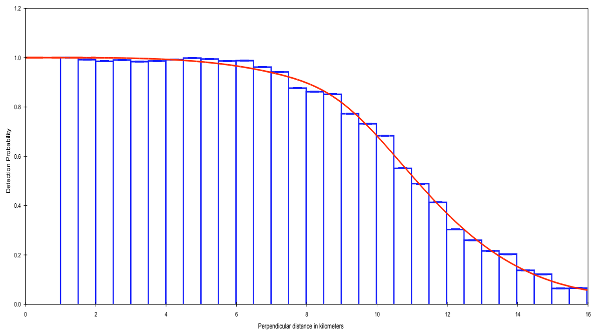
There are six models applied to the left truncated data, the first three models (M_1, M_2, M_3) using a Normal detection function, but with three different adjustment terms, while the other three models (M_4, M_5, M_6) using the same three adjustment terms respectively but on a Hazard-rate detection function. The results obtained for these models, after numerical maximization of the appropriate likelihoods, are shown in Table 3. Coefficient of variances were obtained using the standard point transect variance estimator for six models and a calculation equation to convert these to the sector radar case.

Table 2. The result for the radial method

Model Defn	Adjustment	Model	AIC	D	D CV	P	P CV
Normal	Cosine	M_1	79235	885.95	2.85E-02	0.0697	2.30E-02
	Simple polynomial	M_2	79255	909.96	1.73E-02	0.0678	3.98E-03
	Hermit polynomial	M_3	79312	932.69	5.24E-02	0.0663	1.18E-02
Hazard Rate	Cosine	M_4	79153	886.53	2.86E-02	0.0688	0.023054
	Simple polynomial	M_5	79152	891.75	1.86E-02	0.0693	7.98E-03
	Hermit polynomial	M_6	79152	899.22	0.021131	0.0689	1.28E-02

***D** represent the density of bird, which is calculated using the results of Distance software. **D CV** is the coefficients of variance of the density estimator and **P** is the detection probability of this survey. The models are applied to the data with a left truncation of $1\ km$, one of which indentified as red is the optimal model for the relevant data set diagnosed by AIC statistics.*

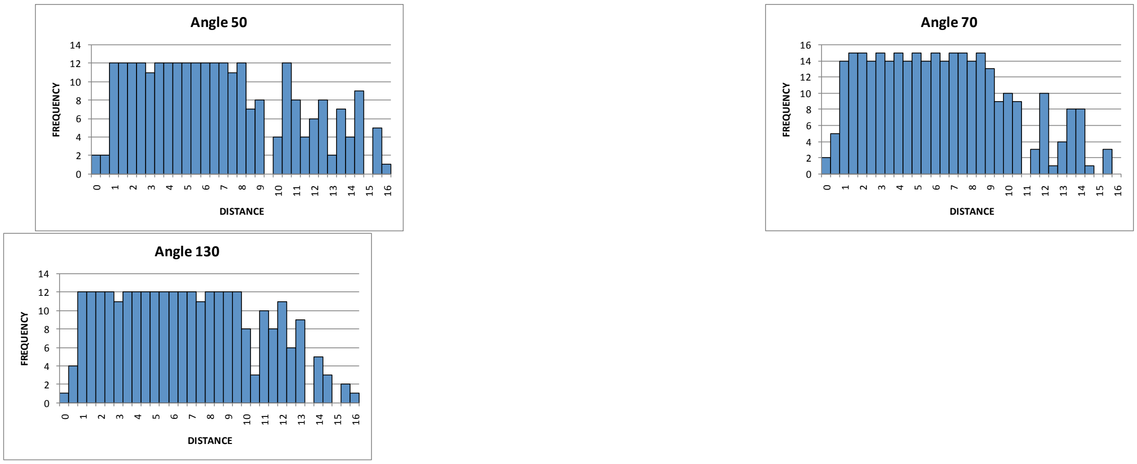
From *Table 2*, we consider model M_5 as the optimal model for the radial data for the AIC statistics is smallest with this model. Hence, the estimation of the bird density is 891, with coefficient of variance to be 0.086. The detective probability function is shown in *Figure 10*, which shows our estimation fits quite well based our primary assumptions.



6 *The plot for the detect function $g(\cdot)$ for the radial method. These two functions are modelled with data set that right truncated at $16\ km$ and left truncated at $1\ km$ based on the equation shown above. And they fit well within these distances.*

In the calculation, we also consider each radial as identically independent distributed detection area. *Figure 9* shows that the abundance of birds detected at each angle fluctuates as the angle changes, but being rough stable for the data combining all angles, which is different with the sector angles. But as the data was selected from radial lines of small and constant width, the horizontal gaps could not be apparent if ignore the errors from selecting procedure (losing the data that on the radial line). It seems that our previous assumptions might be checked out. But, because the radial method and the sector method are two sampling methods generated from a same radar detection image, the nonuniform patterns of the seabird should also exist in the data of the radial method. The histograms of distance with different radial angles showing in *Figure 11* display this sudden booming detection at the middle of distance caused by the nonuniform distribution of seabird and the dissimilar patterns of the detection from each radial line. Like the histogram of distance at angle 50, the abnormal form of the radial detection occurs around $10\ km$. But as the width of the radial line is relevant small, the cumulate detection histogram does not show any abnormal pattern. At this point, the Radial method is better to deal with the radar image than the sector method.

Horizontal Distance



7 *Histograms of distance with radial method on the angle 50, 70 and 130. Though it is not so obvious as that of the sector method, the number of birds makes a sudden growth on the distance between* 10 *km and* 15 *km at these angles*.

However, besides the nonuniform distribution, the size of bird may also generate deviation on the density we estimated. As the process of collecting data on the radial line, the probability to find a small bird on the radial line is smaller than the probability to find a large bird on the radial line at the same distance. This difference in the detection probability (detect a bird on the radial line) caused by the bird size should be considered into the model. There are two modifications to improve the previous model.

I. Condition that the sizes of birds are provided in the survey

From previous discussion, we know that the size of birds may be important in the estimation accuracy, which this type of information is not shown in the original data set. If the data set provided the information about the size of birds, we can modify the radial model with considering the size of birds.

As we discussion in the radial model, the detection density function $f(r)$ can be expressed as,

$$f(r,v)dr = \frac{g(r)2kv}{P_{a\_radial}}\frac{dr}{\varepsilon\pi w^2}$$

The detection density gets smaller not only when the bird is further away from the radar, but also when the size of the bird gets smaller. The detection probability P_{a_radial} in this equation is the probability to detect n birds in the detection area, which sized $v_1, v_2, \dots, v_i, \dots v_n$ respectively. That means that the detection probability is

$$\hat{P}_{a\_radial} = \mathrm{E}\left[\frac{2kvg(r)}{\varepsilon\pi w^2 f(r,v)}\right] = \sum_{i=1}^{n}\left[\frac{2kv_i g(r_i)}{\varepsilon\pi w^2 f(r_i,v_i)}\right]$$

denoting the probability of detecting a bird at distance $r = 0$ is 1 ($g(0) = 1$). The detection density function $f(r,v)$ is also modified using MLE to achieve an estimated $\hat{f}(r,v)$. As defining

$$\mathcal{L}(\theta) = \prod_{i=1}^{n} f(r_i, v_i)$$

Where r_i is the ith recorded distance, and v_i is the ith recorded bird size, $i = 1, \ldots, n$.Express the density function as $f(r, v) = g(r)v/A$, where A is constant with r and v, and can be calculated by $A = \mathrm{E}\left[\int_0^w v \cdot g(r)\ dr\right]$. Or we can simply estimate it by

$$A = \frac{1}{n} \cdot \sum_{i=1}^{n} \left\{ v \int_0^w g(r)\ dr \right\}$$

Then log-link likelihood function equals to

$$l = \log_e[\mathcal{L}(\theta)] = \sum_{i=1}^{n} \log_e \left[\frac{g(r_i)(v_i)}{A} \right] = \sum_{i=1}^{n} \{\log_e g(r_i) + \log_e(v_i)\} - \mathrm{n}\log_e A$$

To find the parameters of the density function is to find the numeric values of these parameters to make the log-link l smallest under the condition of given data. The density estimation is the same with the previous radial method with probability P_{a_radial}.

$$D_{radial} = \frac{\mathrm{E}(n)}{a \cdot P_{a\_radial}}$$

The covariance of the density is the same;

$$\widehat{Var}\left(\widehat{D}_{radial}\right) = \widehat{D}_{radial}^2 \left\{ \frac{\widehat{Var}(n)}{n^2} + \frac{\widehat{Var}\left(a \cdot \hat{P}_{a\_radial}\right)}{\left(a \cdot \hat{P}_{a\_radial}\right)^2} \right\}$$

II. Condition for the birds with hierarchical structure.

However, if we still cannot get the information for the size of birds that are detected, we cannot use the first modified model, but we can build the model with considering the size of the birds in another way. If we simply assume there are two main sizes of birds in this area, one is *"big"* (v_{large}), one is *"small"* (v_{small}). And the ratio of the *"big"* birds against the *"small"* birds consists stable during the survey time and across the survey area. And we set the percentage of the *"big"* birds in the whole population of birds as H. We make an assumption that within the distance r_0 of all types of birds can be detected, but if the distance is over r_0, then only *"big"* birds can be captured by the radial method, where r_0 is a given distance.

So the detection density function $f(r)$ represents probability to detect a bird at distance r on the condition that there are n birds are totally detected in the survey area. This probability is;

$$f(r) = \begin{cases} \dfrac{2kg(r) \cdot [Hv_l + (1 - H)v_s]}{\varepsilon\pi w^2 P_{a\_radial}}, r \le r_0 \\ \dfrac{g(r)2k(v_l)}{\varepsilon\pi w^2 P_{a\_radial}}, r > r_0 \end{cases}$$

The detection density gets smaller not only when the bird is further away from the radar, but also when the size of the bird gets smaller. That means the detection probability is

$$\hat{P}_{a\_radial} = \mathrm{E}\left[\frac{2kg(r) \cdot [Hv_l + (1 - H)v_s]}{\varepsilon\pi w^2 f(r)}\right] + \mathrm{E}\left[\frac{g(r)2k(v_l)}{\varepsilon\pi w^2 f(r)}\right]$$

denoting the probability of detecting a bird at distance $r = 0$ is 1 ($g(0) = 1$). The detection density function $f(r)$ is also modified using MLE to achieve the most likelihood function $\hat{f}(r)$. As defining

$$\mathcal{L}(\theta) = \prod_{i=1}^{n} f(r_i)$$

Where r_i is the ith recorded distance, $i = 1, \dots, n$. The density function can be written using a constant$A$

$$f(r) = \begin{cases} \dfrac{g(r) \cdot [Hv_l + (1-H)v_s]}{A}, r \le r_0 \\ \dfrac{g(r)(v_l)}{A}, r > r_0 \end{cases}$$

As $\int_0^w f(r) = 1$, then

$$A = [Hv_l + (1-H)v_s + L] \int_0^{r_0} g(r)\ dr + (v_l + L) \int_{r_0}^{w} g(r)\ dr$$

Then log-link likelihood function equals to

$$l = \log_e[\mathcal{L}(\theta)] = \sum_{i=1}^{n} \log_e \left[\frac{g(r) \cdot [Hv_l + (1-H)v_s]}{A} I(r - r_0) + \frac{g(r)(v_l)}{A} I(r_0 - r) \right]$$

To find the parameters of the density function is to find the numeric values of these parameters to make the log-link l smallest under the condition of given data. Then use these values we can get an estimation of P_{a_radial}, by which we can calculate the bird density in this area;

$$D_{radial} = \frac{n}{k \cdot a \cdot P_{a\_radial}}$$

2. Summary

The density estimated by the sector method is 950 objects per square kilometre, which is larger than that by the radial method—891 objects per square kilometre. These two results are roughly the same as they are generated from the same global radar detection image, but slightly different because of two different sampling methodologies they use. Both the results are showing the cumulated density of birds for four month in this area. It reflects more about the average movements of the birds for long time.

Sector method is more like point transect, providing the data from sector areas of different angles. It is more appropriate to use when the birds spread in the survey area with uniform or other given type of distribution along the radius lines staring from the detection centre. In real case, this method has disadvantages on its sensitive reaction to the nonuniform distributed density. The bird abundance usually fluctuates along with the offshore distance in the field study. Even if this fluctuation follows a certain pattern, as the detection area at distance r gets larger with the increases of the distance, the density fluctuation greatly dissimilates the density distributions for different sectors, and it is hard to numeric calculate this dissimilarity. Therefore, the density calculated by the sector method is very sensitive to the angles picked up from the global image, and has large variance between sectors. However, we can use this dissimilarity to discover the habitat regions for the seabirds (*Figure 6*). Unlike the sector method, radial method gives a relative stable estimator of low variance. Though the density fluctuation still exists in the global detection area (*Figure 11*), by setting the width of sampling spline to be small enough, the data collected as a whole would not be influenced much by this fluctuation (*Figure 9*). So the radial method is better to use for calculating and estimating the average bird abundance from a radar detection image.

Besides the scientific interest in the quantification of bird density, there is an increasing need to quantify bird movements to assess the seabird habitats. For the two models are built to calculate the cumulated density of the detection area using the global image combining from four month detection, both of these methods would show the birds living patterns during the four month on this sea area by considering data separately with different angles. And the cluster of birds is more recognizable in the sector method, from which a general birds living and flying regions can be detected. With the sector method, the distribution of birds spreading in the survey area can be presumed, tested and quantified, which can be then used in the radial method to give an estimation of the bird density considering a horizontal nonuniform distribution of seabirds. This estimation would be less biased than those built in this paper.

With the sector method, the size of the bird slightly influences the probability of detection, being different with the radial method where the size of bird strongly affects the results. The probability to detect a bird on the radial line at certain distance differs with the size of birds. A modified model with considering the impact of bird size is built in this paper.

To subsample this huge amount of data, the sample was picked up belonging to systematically defined transects from the whole image. This may lead to temporal gaps in the horizontal coverage. Though the collecting method (i.e. resulting in the whole image) yields with no horizontal gap, posterior horizontal gaps would appear caused by the nonuniform distribution of birds in the detection area, especially for the case of sector method. Radial method by the current setting would be affected slightly by these gaps in the results judging from *Table 2*. For this method, temporal gaps would usually be caused by birds crossing the radial line without being detected, which is related to bird size. So information about the size of bird or a model considering the hierarchical structure is requisite for this study.

Censusing seabird from coastal areas requires reliable estimates of bird numbers and the distances of the birds from the coastline. Radar method is a very effective way to quantify these two variables, while different sampling methods would lead to different results. The diversity of observation location and research point may change the optimal method we should use. Scientific studies have usually paid considerable attention to appropriate recording and cautious interpretation of radar data. However, a proper sampling method to pick up sufficient information from the radar image for estimating bird abundance is essential. This paper introduces two methods: sector method and radial method. With comparison of the two estimations, the radial method would be the optimal one, especially with additional consideration of the bird size.

Reference

[1] Bruderer B. and Steidinger P.. Methods of quantitative and qualitative analysis of bird migration with a tracking radar. *Animal Orientation and Navigation* 1972; 151–67.

[2] Desholm M., Fox A. D., Beasley P. D. L. and Kahlert J. Remote techniques for counting and estimating the number of bird-wind turbine collisions at sea: a review. 2006; **148**: 76–89.

[3] Baird R.W., and Buckhart S. M.. Bias and variability in distance estimation on the water: implications for the management of whale watching. Paper SC/52/WW1, IWC Scientific Committee, Adelaide, Australia. 2000.

[3] Buckland, S.T., T., Aderson D. R., Burnham K. P., Laake J. L., Borchers D. L., and Thomas L. *Introduction to distance sampling, estimating abundance of biological populations*. Oxford University Press, Oxford, UK. 2001.

[4]. *Advanced distance sampling*. Oxford University Press, Oxford, UK. 2004.

[5] Burnham, K. P., and Anderson D. R.. *Model selection and multi-model inference: a practical information-theoretic approach*. Springer-Verlag, New York, NY. 2002.

[6] Camp, R. J. Measurement errors in Hawaiian forest bird surveys and their effect on density estimation. *Hawaii Cooperative Studies Unit Technical Report HCSU-005*, University of Hawai'i at Hilo, Hilo, HI. 2007.

[7] Camphuysen, C. J., Fox A. D., Leopold M. F., and Petersen I. K. 2004. Towards standardized seabirds at sea census techniques in connection with environmental impact assessments for offshore wind farms in the UK. Royal Netherlands Institute for Sea Research and the Danish National Environmental Research Institute, Crown Estate Commissioners, London, UK.

[8] Chen, S. X. Measurement errors in line transect surveys. *Biometrics* 1998; **54**: 899–908.

[9] A. Cowling. Measurement errors in line transect surveys where detectability varies with distance and size. *Biometrics* 2001; 57: 732–42.

[10] Schmaljohann H., Bachler F.L, E. Quantification of bird migration by radar—a detection probability problem. 2008; **150**:342-55

[11] Maters, M.. Radar technology applied to the study of seabird migration across the Strait of Gibraltar. Ph.D. dissertation. University of Cadiz, Cadiz, Spain. 2009

[12] G. M.Arroyo, A.Rodrigues, and D.Cuenca. Calibration of visually estimated distances to migrating seabirds with radar measurements. *Journal of Field Ornithology*. 2010; **81**(3): 302-9.

[13] Marques T. A., Buckland S. T., Borchers D. L., Tosh D., and Mcdonald R. A. Point Transect sampling along linear features. *Biometrics*. 2010; 66, 1247–55.

[14] Thomas, L., Buckland, S. T., Rexstad, E. A., Laake, J. L., Strindberg, S., Hedley, S. L., Bishop, J. R. B., Marques, T. A., and Burnham, K. P.. Distance software: Design and analysis of distance sampling surveys for estimating population size. *Journal of Applied Ecology*. 2009